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The Plato Mind

Mathematics in Japan and China

Vector Analysis and Trigonometry

Computation of the Simple Interest Rate in Installment Purchases

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This Journal is dedicated to the following aims:

1. THROUGH PUBLISHED STANDARD PAPERS ON THE CULTURE ASPECTS, HUMANISM AND HISTORY OF MATHEMATICS TO DEEPEN AND TO WIDEN PUBLIC INTEREST IN ITS VALUES.
2. TO SUPPLY AN ADDITIONAL MEDIUM FOR THE PUBLICATION OF EXPOSITORY MATHEMATICAL ARTICLES.
3. TO PROMOTE MORE SCIENTIFIC METHODS OF TEACHING MATHEMATICS.
4. TO PUBLISH AND TO DISTRIBUTE TO THE GROUPS MOST INTERESTED HIGH-CLASS PAPERS OF RESEARCH QUALITY REPRESENTING ALL MATHEMATICAL FIELDS.

The Plato Mind

The following citation is from David Eugene Smith's History of Mathematics: "Speaking of the use of mechanical appliances in geometry, Plutarch remarks upon 'Plato's indignation at it and his invectives against it as the mere corruption and annihilation of the one good of geometry, which was thus shamefully turning its back upon the unembodied objects of pure intelligence.' That Plato should hold the views here indicated is not a cause for surprise. The world's greatest thinkers have always held it. No man ever created a mathematical theory for practical purposes alone."

It is a far cry from this lofty level of the Greek geometer in a machineless age to the plane of present-day commercial valuation of mathematics. In this era of material power and machines, only application and utility seem to be dominant motives in training. In high school the administrative slogan is becoming: Require of the student only what mathematics he shall need to earn a living. Consideration of any other sort of need is taboo. In college, through the door of student-option privilege troop hundreds of students who, like broken field runners avoiding interference on a football field, race to their diploma job insurance, deftly dodging every semblance of mathematics. Anyway, what has an unembodied object of intelligence to do with job-holding?

And something of the same sordid unintellectual motive has crept into graduate schools. When will the Plato attitude begin to be dominant? Only when more of our graduate groups shall think and pray less for higher degrees and more for learning for learning's sake, more for science for the sake of science, more for *mathematics* for the sake of *mathematics!*

Mathematics In Japan and China

JULIA T. COLPITTS
Iowa State College

I was very fortunate to be in Japan and China when both public schools and colleges were in session. In Korea, the colleges had closed, but I visited some high schools.

In Japanese towns, I usually stayed at the homes of friends. When plans were being made for my entertainment, I always said "I want to spend some time visiting mathematics classes." I am sure that my friends often thought it strange that one would come to a country noted for fine scenery, interesting temples and historic places and ask to spend time in mathematics classes. However, they always kindly took me to the schools and I found them intensely interesting.

When you go to visit a school in Japan you are probably asked either to take off your shoes and put on slippers or you are given some cloth covers to put on over your shoes. You are certain to be introduced to a class. In that case, you go forward, make a deep bow and each student stands and bows.

The Japanese language is, as you know, very difficult but it is easier to learn to count in Japanese than in English. In order to count to one hundred, one has only to learn the first ten numbers and the law of formation of the others—the first ten are ichi, ni, san, shi, go, roku, shichi, hachi, ku, ju. The next nine numbers are formed by adding the first nine of these in succession after "ju." Thus eleven is "ju ichi," twelve is "ju ni," etc. Twenty is "ni ju." That is two tens. "Ni ju ichi" is 21, etc. "San ju" is 30 and each multiple of ten is formed in the same way.

In Japan, the teachers whose classes I happened to visit could not speak English very well and I could not talk with them much outside of class—but they were always very courteous and when I asked them if I could buy one of their text books, they usually presented me with one.

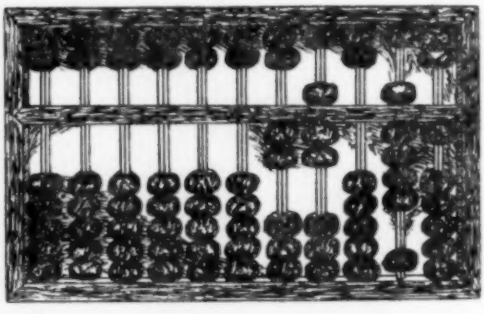
I visited all kinds of mathematics classes from the primary up through the college. In Japan, there are six years of work in primary school, five in high school or middle school as they call it, three in college and three in the university. Boys and girls are together in primary but they have separate high schools. Colleges and universities are usually not coeducational. In China, practically all universities are coeducational.

In the second year primary class which I visited, the teacher at the beginning gave oral questions in addition and subtraction. Then some of the children went to the board and carefully explained examples in subtraction for instance how 25 is taken from 82.

In their text book there were written examples like this. "If one article costs 45 sen and another 8 sen, how much change would a person get from a yen? (The yen corresponds to our dollar and is 100 sen). How much change from 50 sen and 10 sen?

In Japan and Korea, they usually give a little work on the abacus either during the last part of the primary or the first part of the high school. It is not required by law but they feel that it is well to learn a little about it since it is used so much in business. I think that they teach it in all schools in China. In Korea I saw two or three addition lessons on the abacus and the children seemed very interested.

A Chinese abacus is called "swanpan" meaning "reckoning board," and a Japanese abacus is called "soroban." The word abacus is never used in these countries.



* A CHINESE ABACUS

In the Chinese swanpan there is a partition across and on each wire there are five balls below the partition and two above. Each one below counts as one and each above as 5 when they are next to the partition. The Japanese soroban is the same excepting that it has only one above. This is probably an improvement since two fives do not seem necessary. Also the soroban has buttons with sharp edges. They think that the fingers can hold these better without slipping.

The swanpan was introduced into China about the twelfth century. One story about the introduction into Japan is rather interesting. In the last part of the sixteenth century, the famous general Hideyoshi having succeeded in getting Japan under his control decided to make his court an intellectual one and sent a man by the name of Mori over

* From David Eugene Smith's History of Mathematics.

to China to bring back mathematical knowledge in which Japan was backward. Some say that Mori brought back the abacus. David Eugene Smith thinks that through business men it probably came into Japan before this, but Mori was an expert in its use and did much to make it popular. He advertised himself as the leading instructor of division in the world.

Addition, subtraction, multiplication, division and extraction of square and cube roots can all be performed on the abacus. In banks, shops and all business places, the abacus seems to be used entirely. Some books state that the people in these countries became so expert with the abacus that they can carry through long computations more rapidly than an expert computer can in writing. I do not believe that this is true when the computation involves division. The superintendent of schools in a city in China told me that they feel that addition and subtraction can be performed more quickly on the abacus than on an adding machine but that multiplication and division can be done more quickly by other methods. Most of my experience with the abacus was waiting in banks until the banker had decided how many yens I should receive for ten dollars when the yen was perhaps $30\frac{1}{8}$ or $31\frac{1}{4}$ cents. I had time to work the problem and check it a few times before the abacus operation was completed. One disadvantage in its use is that they make mistakes occasionally and can check only by performing the work again. One day I called attention to a mistake of ten sen in my amount. The man agreed with me after going through all the long work of division again. If his work had been on paper, he could probably have seen his error immediately.

All high schools have entrance examinations and the best schools can select their students since they have many more applicants than they can admit. In one mission school in Tokyo, there were last year 1250 applicants to the boys' high school and only 250 admitted, while for the girls' high school there were 800 applicants of which 200 were taken. In this way many high schools have very select students.

I thought sometimes that children in the lower grades did not seem very far advanced in mathematics. There is every reason why this might be true. You know they have characters to represent words. Altogether in their school life, they learn about 10,000 characters. The study of these takes up very much time. A Chinese girl told me that when she started to school, she learned 5 a day and later the number was increased to ten a day.

Perhaps a Korean student has the greatest language difficulty. The Koreans still speak the Korean language and study it in schools. They are required also to study Japanese which is very different and

the Japanese language is used in most of the classes. They thus have two difficult languages at the beginning. The head of a boys' high school in Korea gave me a list of subjects that a boy must take before graduating from high school: arithmetic, algebra, geometry, trigonometry, morals, civics, Korean, Japanese and English languages, history (Japanese, Oriental and Western), physical geography, botany, chemistry, physics, physiology and general biology, commercial bookkeeping, music (vocal and instrumental), Chinese penmanship, English penmanship and mechanical drawing. As in Japan they have six years of primary and five of high school. Fortunately the Koreans are very bright and eager to learn and are naturally good linguists.

In spite of the language difficulties, at the end of the eleventh grade in Japan, the students have had more mathematics than our students in general have had at that time.

In the boys' high schools in Japan, they have algebra including logarithms, progressions, etc. plane and solid geometry and trigonometry. I think that the girls' high schools usually omit trigonometry. Girls usually take many subjects to prepare them for home life such as etiquette, flower arrangement, cooking and sewing. Sometimes, there are two classes in each of these subjects, foreign and Japanese. English is taught in high schools in Japan although many of their graduates do not seem very familiar with it.

When I saw the board work in the high school classes which I visited in Japan, I almost thought I was back in America because they use our letters and symbols but the explanations which were in Japanese did not sound very familiar excepting the words plus, minus, root and equals. At the beginning of the first algebra class I visited, the teacher was explaining rationalization of fractions. At first he put $1/\sqrt{5}-\sqrt{3}$ on the board and explained very carefully what one must multiply numerator and denominator by. Then he took a fraction with radicals in both numerator and denominator. The next class I visited had a lesson on progressions and one might think the examples were taken right from our books.

At the beginning of a class in trigonometry all the fundamental formulas $\sin^2 \alpha + \cos^2 \alpha = 1$ and so on, were written on the board just as we write them. During the recitations, formulas for functions of $90° - \alpha$, etc., were developed, also students were taught to interpolate in using logarithm tables. All the letters and symbols on the board were in English. In geometry, the figures had English letters and the proofs were like ours excepting a Japanese word at the beginning which I felt sure was "given" and sometimes one near the

end which I translated as "likewise." The reasons were in Japanese.

As a general thing in Japan, the classes are large, sometimes fifty or sixty, and the room often has blackboard only in front. The teacher does most of the work and the students copy it in notebooks. The Americans in Japan feel that it is not a very satisfactory method, that it would be better for the student to do more of the work. However, when the students did put work on the board it was always carefully and accurately done.

In Japan I visited a number of women's colleges but the only one doing much work in mathematics was the Christian College in Tokyo where I spent four or five hours in classes. Their work seemed to be of high standard and the work put on the board by the students was well done.

I had hoped to visit classes in the Imperial University in Tokyo and a friend there wrote to the President asking if he could bring me. The President replied that it was not customery to have visitors as they felt it disturbed classes but he invited us to lunch at the University restaurant. When we arrived, clerks with very deep bows ushered us into the President's office. Although it was almost time for lunch we were at once served with green tea. Not to offer tea or some drink in Japan would be almost as discourteous as not to ask anyone to be seated in this country. The President had invited to meet me, two mathematics professors and two other professors who spoke English. The mathematics professors could not speak English very well but after lunch they showed me the mathematics library. They had the standard books and magazines in English, French and German in addition to a large number of Japanese books.

In China, I visited the universities more than the high schools. The professors all spoke English very well so that I could talk to them about their work. In high school, the students take college algebra, trigonometry and analytics. They must take two examinations to graduate from high school. First they take one from their own school. Then there is a uniform government examination which is given at one place in each province. If this second examination is not passed they can neither graduate from their high school nor take the university examinations. They classify high schools according to the number passing this uniform examination.

Admission to universities is by examination and each university prepares its own entrance examination. Some are much more difficult than others. I think all the universities I visited admitted only a small number of their applicants. One university has a special examination

for science students in addition to the entrance examination. Those who fail absolutely in mathematics in this are advised or required to take another course. Those who fail but not so badly are allowed a half year or year to make it up.

The government university of Jow Toong in Shanghai has very high standing in mathematics and engineering. They give very stiff entrance examinations and get a very select group. From one thousand applicants, they admit only two hundred. The President told me that some of the faculty favored lower standards so as to admit more but he felt that since it was a government supported institution, it was only fair to take the best students and train them for government positions. In this university, mathematics majors are required to take calculus in the first year, advanced calculus, differential equations and solid analytics in the second, modern algebra, modern geometry and functions of a complex variable in the third while the fourth is a continuation of the third with theory of numbers and theory of groups. Theoretical physics, mechanics, electricity and magnetism and modern physics are also required. They must take German three years so that they can read scientific books. Physics and chemistry students take considerable work in mathematics.

In all the universities I visited in China, the mathematics textbooks were in English. Granville's Calculus is very popular but not used altogether. In Jow Toong University the lectures are usually given in English, the examination questions are in English and usually the papers. Please do not think I am advertising if I make a personal reference here. Professor Pau of the University of Nanking speaking of text books said they were using an analytics by Roberts and ———. He could not think of the other name and evidently had not understood mine when we were introduced. So I filled in the missing name and told him that I was the other author. He was so pleased that he got up and shook hands over again. He was extremely kind to me during my visit. He took me to visit mathematics classes in his own university and one day he went with me to Central University in Nanking. He remained while I visited the mathematics classes there—then took me to lunch at a Chinese restaurant.

The mathematics professors that I met seemed very anxious to bring up mathematical standards and then try different methods in class. One professor in the University of Nanking said that in addition to the three hour class room work in analytics which was a three hour credit course he gives a two hour examination each week. The papers are corrected and returned to the students and general mistakes are

discussed. He does this in place of having them hand in home work. He feels that the results have been very satisfactory.

The universities find that some of the students are poorly prepared in high school mathematics and science. Last year a few universities had two weeks' summer school to help the high school teachers by giving them better methods and so on. The government thought it such a good thing that this last summer they asked sixteen universities to each give four weeks' summer school to help high school teachers of mathematics and science. Some professors are giving their time without pay in these summer schools. Two professors from different universities in Nanking were talking to me about these schools. I remarked that I supposed they received extra pay. They said "no" but one of them added that in his university they were offered pay but felt that they ought to do that much to help.

In China, there is a recent law that in order to teach mathematics in high schools, one must be a university graduate with either a major or minor in mathematics. Unfortunately, at present there are not enough trained teachers to enable this law to be enforced. Trained teachers in mathematics are very much in demand. The head of the mathematics department at Ginling, a college of high rank for women but of limited attendance, said he had had ten requests for mathematics teachers but there were only two mathematics majors in the senior class.

The Americans in China as well as in Japan feel that the teachers do too much of the work. The students copy the work but do not always understand it and do not have much opportunity to ask questions. They have almost a lecture system in high school. The amount of blackboard is usually limited so students cannot do much board work in class.

In surveying the field of mathematics in China, we find that the Americans feel that the methods of teaching are not the best as the teacher does too much of the work—but the Chinese are extremely fortunate in having a minister of education who realizes the value of mathematics, who arranges special summer courses in order to raise the standards of the high school teachers and who realizes the importance of having mathematics teachers thoroughly trained in mathematics.

Vector Analysis and Trigonometry

By W. E. Byrne

The exclusive retention of the concept of distance as learned in elementary geometry prevents many students from acquiring the important concept of algebraic measure of a segment referred to an oriented axis. It is the object of this article to put into evidence some discussions of trigonometry which belong to vector analysis rather than to school geometry. To do this it will be necessary to recall several definitions.

Definition 1. The algebraic measure of a vector AB with reference to an oriented line L parallel to or containing the segment AB equals the length of AB with a positive or negative sign according to whether the displacement from A to B is in the positive or negative sense of L. In figure 1 the algebraic measure of AB is positive and that of BA negative, the positive sense of L being indicated by an arrow.

Figure 1.

Definition 2. The orthogonal projection of a vector AB on a line L (oriented or not) is a vector A′B′ .A′, B′ are the feet of the perpendiculars from A, B, respectively, to L, that is, the intersections of planes through A, B perpendicular to L with L.

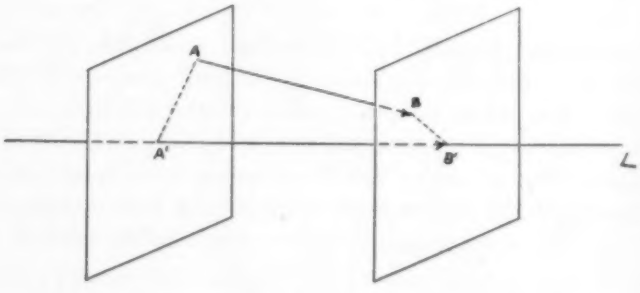


Figure 2.

Definition 3. If A, B lie on a line L, the vector AB is said to be carried by L.

If use is made of the definition of cosine there appears

Theorem 1. The algebraic measure l′ of the projection on an oriented

line L′ of a vector V carried on an oriented line L equals the algebraic measure l of V with respect to L multiplied by the cosine of the angle between the oriented lines L and L′.

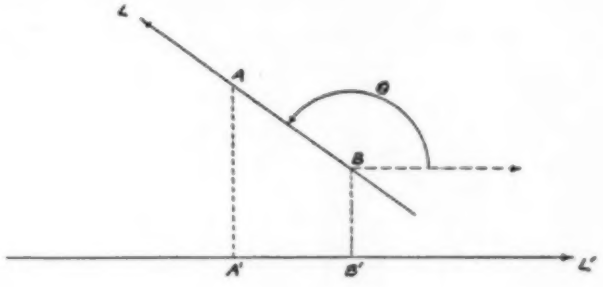


Figure 3.

(1) $\quad l' = l \cos \theta = l \cos (\widehat{L, L'})$

Note that L, L′ may be skew lines.

Corollary. The sine and cosine of an angle θ are the algebraic measures of the projections of a unit vector OM on the oriented lines Ox, Oy indicated in figure 4.

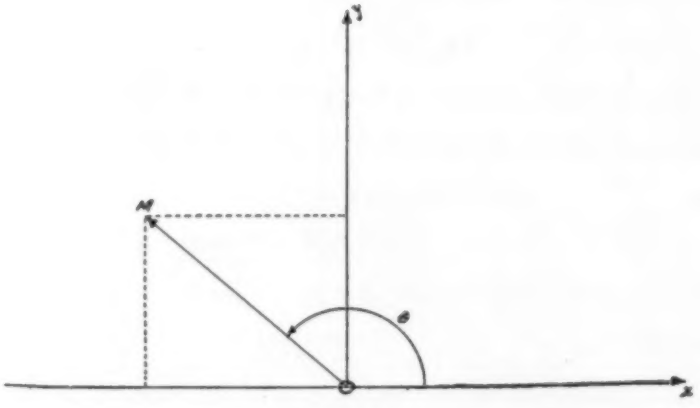


Figure 4.

Special cases of the addition formulas may be derived immediately from the previous considerations.

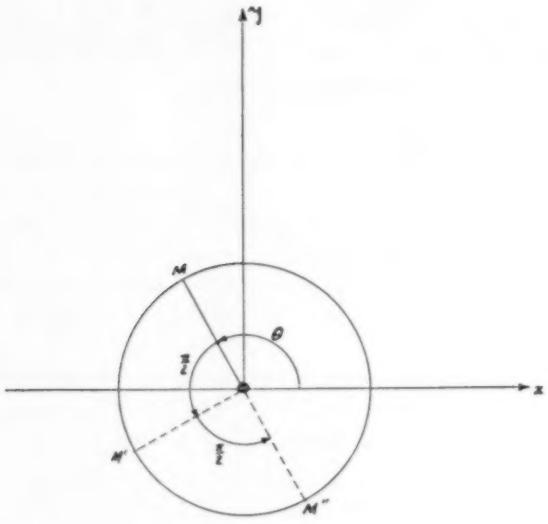


Figure 5.

$$(2) \qquad \begin{cases} \cos(\theta \pm \Pi) = -\cos\theta \\ \sin(\theta \pm \Pi) = -\sin\theta \end{cases}$$

Formulas (2) follow from Theorem 1 and its corollary.

$$\mathrm{Ox}, \overset{\frown}{\mathrm{OM}} = \mathrm{Oy}, \overset{\frown}{\mathrm{OM}'}$$

Hence the algebraic measure of the projection of $\overrightarrow{OM}$ on Ox equals the algebraic measure of the projection of $\overrightarrow{OM}'$ on Oy, i. e.

$$(3) \qquad \cos\theta = \sin(\theta + \Pi/2)$$

(3) is true for θ arbitrary. If θ is replaced by $\theta + \Pi/2$,

$$(4) \qquad \cos(\theta + \Pi/2) = \sin(\theta + \Pi) = -\sin\theta$$

by application of (2).

The introduction of the vector sum of two vectors in connection with the preceding formulas permits the demonstration of the addition formulas.

Definition 4. The vector sum of two vectors $\overrightarrow{V}_1$, $\overrightarrow{V}_2$ is the vector obtained by placing the initial end of $\overrightarrow{V}_2$ at the terminal end of $\overrightarrow{V}_1$ and joining the initial end of $\overrightarrow{V}_1$ to the terminal end of $\overrightarrow{V}_2$.

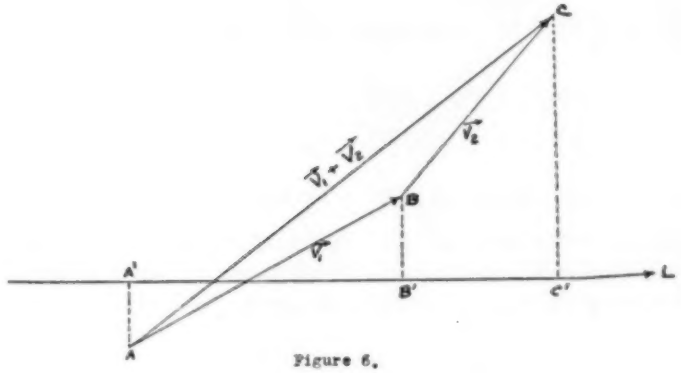


Figure 6.

A consequence of this definition is the

Theorem 2. The algebraic measure of the projection of $\vec{V}_1+\vec{V}_2$ on L equals the sum of the algebraic measures of the projections of $\vec{V}_1$ and $\vec{V}_2$ on L.

This may be applied to the addition theorems as follows:

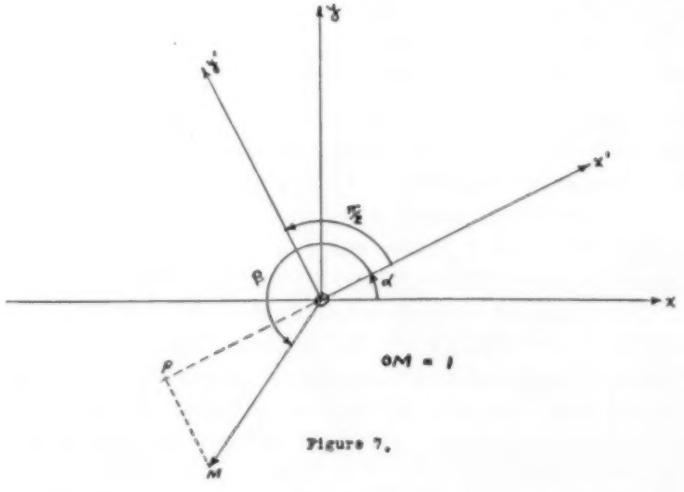


Figure 7.

Introduce auxillary axes as indicated in figure 7 and apply theorem 2 to the projections of $\vec{OM}$, $\vec{OP}$, $\vec{PM}$ on the oriented axes Ox, Oy to find:

Proj. $_{Ox}\vec{OM} = \cos(\alpha+\beta)$ by corollary of theorem 1

$$= \text{Proj.}_{Ox}(\overrightarrow{OP} + \overrightarrow{PM})$$

$$= \text{Proj.}_{Ox}\overrightarrow{OP} + \text{Proj.}_{Ox}\overrightarrow{PM} \text{ by theorem 2}$$

$$= \cos\beta \cos\alpha + \sin\beta \cos(\alpha + \pi/2) \text{ by theorem 1.}$$

since $\overrightarrow{OP}$ may be considered as a vector of algebraic measure $\cos\beta$ with respect to the oriented axis Ox', and $\overrightarrow{PM}$ may be considered as a vector of algebraic measure $\sin\beta$ with respect to the oriented axis Oy'. Formula (4) reduces the above results to

$$\cos\beta\cos\alpha - \sin\beta\sin\alpha$$

Hence

$$(5) \qquad \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\beta\sin\alpha$$

The same argument applies to the addition formula for sines. It is only necessary to project on Oy instead of on Ox.

The vector analysis point of view which has just been somewhat sketchily outlined is of importance for later applications as well as for a clearer understanding of the interpretation of signes. The usual proof of the addition formulas does not make use of the full generality of the definition of sines and cosines. The justification of all results hinges on theorem 1, which may be verified in detail.

Might there not be given in elementary mathematics, especially, a classification of material according to the methods most favorable to a clear, concise development? That would lead to another problem: to ascertain the mathematical point of view which permits the most direct presentation of a given subject. The discussion of tangent lines in analytic geometry is an example of this problem: some authors insist on giving special artifices for each conic section while others use the methods of calculus for all the conics. Again, suggestions have been made that area and volume concepts of plane and solid geometry be developed from the point of view of calculus. It would seem that there is considerable room for development in modernizing even elementary subjects.

Computation of the Simple Interest Rate In Installment Purchases*

By H. E. STELSON
Kent State College, Kent, Ohio

Although it is well known that the simple interest rate is smaller than the compound interest rate for a series of payments on a given debt extending over periods less than one year, the magnitude of this difference is not usually recognized. Moreover, in view of the current practice of charging simple interest in transactions in which the time is less than a year it seems more logical to inquire the simple interest rate than the compound interest rate.

It is the purpose of this paper to develop some approximation formulas for computing the simple interest rate. The following example will be used to illustrate the theory which will be presented. A gas range sells for \$163 cash or \$15 down and \$20 per month for nine months. Find the interest rate which will make the installment plan equivalent to the cash price.

By interpolation in an interest table the effective compound interest is found to be 62.07%. The compound interest as computed by Todhunter's† Approximation formula is 62.05%. By a series of interpolations the simple interest may be found to be 54.62% which is 13.4% less than the compound interest.

If we let L = periodic interest rate, m = number of payments, V_0 = cash price, D_0 = down payment and R = the periodic payment, then the problem involves the solution of the equation.

$$(1) \qquad \frac{V_0 - D_0}{R} \equiv K = \sum_{x=1}^{m} (1+xL)^{-1}$$

for L. Expanding the right member and summing the terms as far as cubes, we have

$$(2) \quad K = m - \frac{m(m+1)L}{2} + \frac{m(m+1)(2m+1)L^2}{6} - \frac{m^2(m+1)^2L^3}{4}$$

*Presented at the meeting of the Ohio section of the Mathematical Association of America April 6, 1933.

†Todhunter's Textbook on Compound Interest and Annuities-certain. 1931. P. 192, Formula (5).

Taking only first degree terms we have the formula

$$L = \frac{2(m-K)}{m(1+m)} \tag{3}$$

which is also the first approximation to the compound interest rate. For the example, the yearly rate $12\,L = 42\frac{2}{3}\%$. This result is decidedly too low but it is frequently used as correct.* The terms of Formula (2) converge so slowly that the result is not satisfactory until the third degree terms are retained. This computation would be too laborious.

However, if we replace L by $L'+P$ where L' is close to L then L can be expressed in the explicit form,

$$L = \frac{\dfrac{12(m-K)}{m(m+1)} - 2\,L'^2(2m+1)+6\,L'^3m(m+1)}{9\,L'^2m(m+1)-4\,L'(2m+1)+6} \tag{4}$$

For example the, if $L' = .05$ then $12\,L = 52.78\%$ which is an error of 1.8%. Formula (4) may be extended and written in the form

$$L = \frac{m-K-C_2L'^2+2C_3L'^3-3C_4L'^4}{C_1+2C_2L'+3C_3L'^2-4C_4L'^3} \tag{5}$$

where C_r equals the sum of the rth powers of the first m natural numbers. For the example, if $L' = .05$, $12L = 55.26\%$ which is an error of $.6\%$.

If we form a difference table of the terms of the right member of (1), then by means of the relation $\sum_1^n u_r = n_1u_1+n_2\Delta u_1+n_3\Delta u_1 \ldots$ the formula

$$K = \frac{m}{1+L} - \frac{m_2L}{(1+L)(1+2L)} + \frac{2m_3L^2}{(1+L)(1+2L)(1+3L)}\ldots \tag{6}$$

is derived. For the example, if $L' = .05$, $K' = 7.301$; if $L'' = .04$, $K'' = 7.570$ so that by interpolation $12L = 55.55$. An approximation to (6) is given by

$$L = \frac{2(m-K)}{6K+m(m-5)}. \tag{7}$$

*Compare Stone Mallory and Grossnickle's Higher Arithmetic.

For the example $12L = 48\%$.

The simple discount rate is expressed by

$$K = \sum_{r=1}^{m} (1-rd) \tag{8}$$

so that

$$d = \frac{2(m-K)}{m(1+m)} \tag{9}$$

the same as (3). If we use n as the weighted mean time and substitute in the formula $L = \frac{d}{1-nd}$, we have the formula

$$L = \frac{2(m-K)}{K(m+1)} \tag{10}$$

which gives a result of 51.9% for the example which is a fair approximation in view of the simplicity of (10). Formula (10) also is the result of the reciprocal approximation of the compound interest formula. Or again (10) may be derived directly from the simple interest formula $I = PRT$ where I is the penalty, P the remaining cash value and T the average number of months. i. e.

$$mR - (V_0 - D_0) = (V_0 - D_0)\frac{m+1}{2}L$$

which results in (10).

If we substitute $f(x) = \frac{1}{1+xL}$ in the Euler-McLaurin formula,* we have

$$K = \log (1+Lm)^{1/2} - \frac{Lm}{2(1+Lm)} + \frac{L^2m}{12}\frac{(2+Lm}{(1+Lm)^2} \ldots \tag{11}$$

*See An Elementary Treatise on Actuarial Mathematics by Freeman. 1931. Page 300.

This formula gives quite accurate results when used in the form

$$(12)\quad L = L' + \frac{K - K'}{K'' - K'}\,(L'' - L').$$

For the example $L = 54.74\%$ when $L' = 1/20$, $L'' = 1/24$. If $L''' = 1/22$, $K''' = 7.4007$ which shows that L is a little greater than 54.54%. It might be noted that (3) is the first approximation to (12).

On the Algebra of Mixtures

By C. D. Smith
State College, Mississippi

In the field of applied algebra the problem of mixtures has received less attention than it seems to merit. Most all text books have a few problems set up in such a way that a system of equations serve to identify the required values. The uninformed reader is left to conclude that any problem can be solved when the data lead to a system of equations. However the reader is destined to be disillusioned when he meets the following situations:

Given the following mixtures:

1. A solution contains 18% soda and 2% salt.
2. A solution contains 6% soda and 5% salt.

Required a mixture of the two so as to obtain a solution with 10% soda and 3% salt.

1. A substance contains, Sodium 3%; Lime 5%; Potash 10%.
2. A substance contains, Sodium 8%; Lime 2%; Potash 5%.
3. A substance contains, Sodium 12%; Lime 9%; Potash 3%.

Required to mix the three so as to obtain a substance with, Sodium 4%; Lime 3%; Potash 6%.

To prove that the problem fails in general it is sufficient to assume some designated quantity of the required mixture in either of the above problems, (try a required mixture of 100 lbs.). Now it may be useful as well as interesting to consider types of problems involving mixtures which may have a solution.

We begin by stating a simple problem found in the books.

Problem: A solution contains a% salt and a second substance con tains b% salt. Prepare a mixture which contains c% salt-

The problem obviously has a solution for a $\begin{matrix}<\\>\end{matrix}$ c $\begin{matrix}<\\>\end{matrix}$ b.

Next let us consider the problem of two substances which contain known percentages of two given constituents. It may be desirable to so combine them as to obtain a new substance which contains certain amounts of the two constituents. Such a problem would require that a system of equations be obtained such that the solutions are positive. Let us state the problem as follows.:

Problem: One ore contains a_1% copper and a_2% zinc. A second ore contains b_1% copper and b_2% zinc.

Required, to mix an ore which contains c_1 oz. copper and c_2 oz. zinc.

The required equations are:

$$a_1\%x+b_1\%y=c_1,$$

$$a_2\%x+b_2\%y=c_2.$$

Or

$$a_1x+b_1y=d_1, \qquad \text{where } d=100c.$$

$$a_2x+b_2y=d_2.$$

Dividing by the coefficients of x we will represent the system by:

$$x+aky=bn,$$

$$x+ay=b.$$

The required solutions are,

$$x=b(n-k)/(1-k)>0$$

and

$$y=b(1-n)/a(1-k)>0.$$

These conditions are satisfied for

(1) $$1 \begin{matrix}>\\<\end{matrix} n \begin{matrix}>\\<\end{matrix} k.$$

To show the practical value of the method we take a particular problem. Say for example that you are selling ore and you have one which contains 10% copper and 5% zinc and a second which contains 4% copper and 12% zinc. You receive an order for 8 oz. copper and 10 oz. zinc. You wish to know if the two ores can be so mixed as to

fill the order. Assuming an amount x of the first and y of the second the equations are

$$10x+4y=800,$$

$$5x+12y=1000.$$

Here $k=4/10 \cdot 5/12=16$, and $n=80 \cdot 1/200=2/5$, and the order can be filled since $1>n>k$.

We may extend the method to the case of three variables as follows. Assume three mixtures which contain known percents of given substances and require that the three shall be so mixed as to yield certain amounts of the given substances. The problem leads directly to a system of three equations with positive coefficients where positive solutions are required. We shall seek a simple criterion which will guarantee positive solutions of such a system. If a, b, c, are given percents, d the required amount of the substance contained in the mixture, and x, y, z, are the required amounts to be mixed, the equations are

$$a_1x+b_1y+c_1z=d_1,$$

$$a_2x+b_2y+c_2z=d_2,$$

$$a_3x+b_3y+z_3c=d_3.$$

Let us write the system in the form

$$x+ak_1y+bn_1z=cm_1,$$

$$x+ak_2y+bn_2z=cm_2,$$

$$x+ay+bz=c.$$

We then have

$$\begin{vmatrix} 1 & ak_1 & bn_1 \\ 1 & ak_2 & bn_2 \\ 1 & a & b \end{vmatrix} = ab \begin{vmatrix} 1 & k_1 & n_1 \\ 1 & k_2 & n_2 \\ 1 & 1 & 1 \end{vmatrix} \gtrless 0$$

(2) If $k_2+k_1n_2+n_1 \gtrless k_1+k_2n_1+n_2$.

$$\begin{vmatrix} cm_1 & ak_1 & bn_1 \\ cm_2 & ak_2 & bn_2 \\ c & a & b \end{vmatrix} = abc \begin{vmatrix} m_1 & k_1 & n_1 \\ m_2 & k_2 & n_2 \\ 1 & 1 & 1 \end{vmatrix} \gtrless 0$$

(3) If $m_1k_2+m_2n_1+k_1n_2 \gtrless m_1n_2+m_2k_1+k_2n_1$.

$$\begin{vmatrix} 1 & cm_1 & bn_1 \\ 1 & cm_2 & bn_2 \\ 1 & c & b \end{vmatrix} = bc \begin{vmatrix} 1 & m_1 & n_1 \\ 1 & m_2 & n_2 \\ 1 & 1 & 1 \end{vmatrix} \gtrless 0$$

(4) If $m_2+m_1n_2+n_1 \gtrless m_1+m_2n_1+n_2$.

$$\begin{vmatrix} 1 & ak_1 & cm_1 \\ 1 & ak_2 & cm_2 \\ 1 & a & c \end{vmatrix} = ac \begin{vmatrix} 1 & k_1 & m_1 \\ 1 & k_2 & m_2 \\ 1 & 1 & 1 \end{vmatrix} \gtrless 0$$

(5) If $k_2+k_1m_2+m_1 \gtrless k_1+k_2m_1+m_2$.

Combining (2), (3), (4), (5), we have the criterion

(6) $k_1n_2-k_2n_1 \gtrless (k_1+n_2)-(k_2+n_1)$.

To illustrate the method we take the data:

Ore	*Gold*	*Silver*	*Copper*
x	4%	2%	8%
y	6%	10%	2%
z	10%	8%	4%

Require an ore containing, Gold 8-oz., Silver 6-oz., and Copper 5-oz The equations are

$$4\%x+6\%y+10\%z=8,$$
$$2\%x+10\%y+8\%z=6,$$
$$8\%x+2\%y+4\%z=5.$$

We have, $k_1=6/4 \cdot 8/2=6$, $n_1=10/4 \cdot 8/4=5$,

$k_2=10/2 \cdot 8/2=20$, $n_2=8|2 \cdot 8/4=8$.

Criterion (6) gives $=52<-11$, and the mixture is possible.

In the case of more than three variables the criterion is not so simple and it would seem that the value of such a test lies in its simplicity. In practice one would hardly be required to mix a great number of substances.

On the Discriminant of a Conic

By Clifford Bell
University of California at Los Angeles

The common methods of writing the discriminant of a conic involve either a substitution in a formula or the setting up and expansion of a third order determinant. As the formula is difficult to remember and mistakes are easily made in its application, the latter method is more often used. The determinant form of the discriminant is easier to write, but the use of half coefficients for six of its elements requires great care in substituting the proper values from the equation of the conic. To overcome the above difficulties the following method of writing the discriminant is suggested.

Using homogeneous coordinates, write the equation of the conic in the following form,

$$Ax_1^2 + Bx_2^2 + Cx_3^2$$
$$+Dx_2x_3+Ex_1x_3+Fx_1x_2=O$$

where the square terms appear on the first line and the product terms on the second. Furthermore the arrangement is such that under each square term appears the product of the two remaining variables. The same arrangement is used fo rnon-homogeneous coordinates by replacing x_3 by 1.

From the above equation form the rectangular array of numbers.

$$\begin{array}{ccc} A & B & C \\ D & E & F \end{array}$$

by merely writing the coefficients in the order that they appear. The discriminant is then given by four times the product of the numbers of the first row, plus the product of the numbers of the second row, minus the sum of the three terms obtained by multiplying each member of the first by the square of the number just under it. The result in symbols is therefore, $4ABC+DEF-AD^2-BE^2-CF^2$, which agrees with the formula.

It is important to note that this method is independent of the manner of designating the coefficients, for the above rectangular arrangement of the coefficients of the conic is dependent only upon the positions taken by the variables. This becomes immediately evident upon the application of the method to numerical cases.

The Teacher's Department

Edited by

JOSEPH SEIDLIN AND W. PAUL WEBBER

ANALYSIS AND INTERPRETATION OF SURVEY OF PRESENT STATUS OF SECONDARY MATHEMATICS IN THE UNITED STATES

Last spring the following three questions were sent out to forty-nine State Superintendents of Education including the Superintendent of the District of Columbia:

1. Has mathematics as a required subject for graduation from high schools been eliminated in your state?
2. If it has not been eliminated as a requirement, is there a definite prospect that it will be eliminated at an early date?
3. In your judgment, are there good reasons for the hope that mathematics will have increased use in the secondary schools of your state?

In our October and November issues forty-seven replies were published (Only the state superintendents of Colorado and Idaho failed to send in their replies.). The statistically significant part of the replies is tabulated on the next page.

We find that 31 states require at least one year of high school mathematics; 16 states require no mathematics beyond the eighth grade. Six of the former may drop the requirement in the near future; only one of the latter indicates an increase in the use of courses in mathematics. We may reasonably expect, then, that within a year or two the high schools of half the population of the country will be free from state required courses in mathematics beyond the eighth grade. What next?

This survey in itself is not indicative of any general trend. It merely presents the situation as it exists today. Any extrapolative predictions based on this survey must of necessity be highly speculative and biased by facts other than those available in this survey. Other surveys (of this nature) recorded and analyzed for several years would doubtless disclose trends and tendencies in the curricula of secondary schools of intrinsic and significance. Perhaps the several societies of teachers of mathematics may deem it wise and expedient to provide the necessary funds for a more thorough and exhaustive study of the status of mathematics in our schools. An attempt should be made to obtain replies to questions suggested by this survey from the supervising officer of every high school in the country.

States Requiring at Least One Year of Mathematics Beyond the Eighth Grade	*States Requiring no Mathematics Beyond the Eighth Grade.*	*Population (Approximate)*
	Alabama	2,650,000
Arizona		450,000
	Arkansas	1,850,000
	**California	5,700,000
	**Connecticut	1,600,000
Delaware		250,000
Florida		1,500,000
*Georgia		2,900,000
	**Illinois	7,650,000
Indiana		3,250,000
**Iowa		2,500,000
	Kansas	1,900,000
*Kentucky		2,600,000
Louisiana		2,100,000
	**Maine	800,000
Maryland		1,650,000
	**Masschusetts	4,250,000
Michigan		4,850,000
	*Minnesota	2,550,000
	**Mississippi	2,000,000
Missouri		3,650,000
Montana		550,000
**Nebraska		1,400,000
Nevada		100,000
	**New Hampshire	450,000
New Jersey		4,050,000
New Mexico		450,000
	New York	12,600,000
North Carolina		3,200,000
	North Dakota	700,000
Ohio		6,650,000
**Oklahoma		2,400,000
Oregon		950,000
Pennsylvania		9,650,000
	Rhode Island	700,000
	**South Carolina	1,750,000
South Dakota		700,000
**Tennessee		2,600,000
*Texas		5,850,000
**Utah		500,000
Vermont		350,000
Virginia		2,450,000
Washington		1,550,000
**West Virginia		1,750,000
	**Wisconsin	2,950,000
*Wyoming		250,000
*D. C.		500,000
31		71,600,000
	16	50,100,000

* Requirements in mathematics may be increased.
** Requirements in mathematics may be decreased or eliminated.

The most recent "deserter" from the ranks of the states requiring at least one year of mathematics is New York. The newly revised curriculum of the State of New York will bear close watching and study. How will it actually affect the quantity and quality of mathematics offered in the schools of the state? In the schools of New York City? In the schools of other large cities famed for "strong" departments of mathematics? Paragraph 5, page 6, is pertinent to our inquiry:

> "Mathematics is proposed as a constant for all pupils through the eighth year and for the large majority of pupils in some form probably through the ninth year. The place of mathematics at this point, however, is very uncertain. Only as the material used may be adapted to meet the purposes, the aims and the objectives of pupils and in this way serve to strengthen the curriculum offerings, will it function effectively in adolescent growth."

And, one might add, wherever mathematics is taught so as to "function effectively in adolescent growth" it will continue as a "required" subject with without the benefit of state regulations or recommendations. On the other hand, wherever mathematics is taught merely because it is a "required" subject, it were best not only to eliminate such courses but eliminate the teachers as well.

One of the conclusions reasonably safe to reach as a result of this survey is that courses in algebra and geometry, as state required courses for graduation from high school, are "losing ground." This may be due to poor teaching. It is extremely doubtful, however, that poor teaching is monopolized by teachers of algebra and geometry. The "depression" may have something to do with it. Or it may be that the present reigning power of educationalists are "economics" and "culture" minded, even as a generation ago they were "science" minded. We do have our fads even in the principles and philosophies of education. Perhaps "economists" and culture-ists" are at present riding the crest of a wave, thus theoretically at least remaining furthest from the hollow of the "depression."

Educators, who happen to be teachers of mathematics, are concerned not so much with the fate of mathematics, or even that of the rank and file of teachers of mathematics, as with the effect of less and less training in mathematics for more and more growing adolescents. Will the present and future needs of society be better met by a citizenry weak in mathematical training? Will superficial thinking, however beautifully phrased; will non-mathematical principles of economics,

however authoritatively worded; will science, deprived of its mathematical tools and structure; will all these tend to develop an "enlightened" democracy? Such considerations only should decide what constitutes an all-round education in a democracy that hopes to remain a democracy.

Unfortunately there are those in position of authority and influence in the framing of educational policies who are prejudiced against mathematics. In most cases the prejudice dates back to some sad incident in their own first contacts with courses in mathematics. In other words, their prejudice is not so much against a great field of human learning as against some unfortunate element in their own experience with it, perhaps the way it was taught (to them). What they propose, however, is incredibly illogical. It is like eliminating water from the diet of humans because some water is contaminated. Obviously, purifying the contaminated water would be the more logical and certainly the more health-benefitting procedure.

But, perhaps, it is too much to expect logical thinking from people, who, upon their own admission, have had very little training in logical thinking. How about ourselves, —teachers of mathematics?

According to this survey, mathematics is still among the "required" subjects in most of the secondary schools of the country. Shall we merely sit back and watch courses in mathematics thin out and disappear; then jointly mourn their passing? Or shall we bend our joint efforts toward the intrinsic improvement of both the content and the methods of teaching of courses in mathematics from the second grade on, so that even legislators of educational policies may perceive clearly the necessity for training in mathematics as a constant through all primary and secondary schooling.

Let us re-read both the tabular summary of this survey and the actual "replies" as published in the October and November issues.

—JOSEPH SEIDLIN

WHAT IS THE SOLUTION?

On page 66 of the January issue of the *National Mathematics Magazine*, Chas. H. Sisam, of Colorado College, wonders "if the mathematicians should not recommend that the mathematics of the elementary and secondary schools be moved, bodily, a year or two, later in the curriculum" because mathematics is so hard that pupils do not get as much from its study as they should.

Is mathematics hard for pupils because they study it at too early an age, or is it hard because they do not start to study it early enough? I believe that the situation would be greatly improved if, instead of trying to crowd elementary algebra all into the ninth grade, we should begin the study of the subject in the seventh grade and then spread it out through the eighth and ninth grades. Similarly, if instead of crowding all of the difficulties of geometry into the tenth grade, we should begin the study of informal geometry in the seventh grade and then develop the subject continously through the tenth year, we would diminish the pupil's difficulty considerably.

A continuous course in general mathematics beginning in the seventh grade with the emphasis upon mastery at the various levels, and the development of fundamental concepts as the main aims of such a course might help to bring about a renaissance.

In the light of modern thought some of the traditional and now obsolete topics which we have inflicted upon the pupils are not suitable for teaching at any age. Moreover, it is assumed that no matter when a subject is introduced, a good teacher is indispensable.—W. D. REEVE, *Teachers College, Columbia University.*

A PRIORI KNOWLEDGE AND MATHEMATICS

Mathematical science affords us a brilliant example how far, independently of all experience, we may carry our a priori knowledge. It is true that the mathematician occupies himself with objects and cognitions in so far as they can be represented by means of intuition. But this circumstance is easily overlooked because the said intuition can itself be given a priori, and therefore is hardly to be distinguished from a mere pure conception. Deceived by such a proof of the power of reason; we can perceive no limits to the extension of our knowledge. The light dove, cleaving in free flight the thin air, whose resistance it feels, might imagine that her movements would be far more free and rapid in airless space. Just in the same way did Plato, abandoning the world of sense because of the narrow limits it sets to the understanding, venture upon the wings of ideas beyond it, into the void spaces of pure intellect. He did not reflect that he made no progress by all his efforts; for he met with no resistance which might serve him for a support, as it were, whereon to rest, and on which he might apply his powers in order to let the intellect acquire momentum for its progress.

From Immanual Kant's Introduction to the Critique of Pure Reason.

Notes and News

Edited by
I. MAIZLISH

Professor E. T. Bell's retiring presidential address of the M. A. of A., "The Place of Rigor in Mathematics," will be found in the *Monthly* for December, 1934.

Professor Robert Wood Clack, Professor of Mathematics and Astronomy at Alma College, Alma, Michigan, has translated classical Chinese poetry into English in a volume called "From Bamboo Glade and Lotus Pool." It is published by the Banner Press of Emory University.

Harry W. Stinson, Instructor in Mathematics at the University of Maryland, died on December 11, at the age of 47.

Mr. Charles W. Williams (Harvard) has obtained a fellowship at the University of Maryland and is majoring in Mathematics.

The next general meeting of the International Astronomical Union will be held in Paris, July 10-17, 1935.

The Lamme Medal, given each year to the teacher of Engineering who is judged to have done outstanding pioneer work, was awarded at the Ithaca meeting of the Society for the promotion of Engineering Education to Edward Rose Maurer, Professor of Mechanics at the University of Wisconsin.

The university award granted by Rutgers University for distinguished services was presented on September 18, 1934 to A. E. Meder, associate professor of mathematics at the New Jersey College for Women.

Yeshiva College conferred the degree of Doctor of Humane Letters on Professor Albert Einstein of the Institute for Advanced Study at Princeton, at the opening exercises of the college on October 8. Professor Einstein and Professor David Eugene Smith gave addresses on that occasion.

The Louisiana-Mississippi Sections of the Mathematical Association of America and the National Council for Teachers of Mathematics will hold their meetings at Louisiana College, Pineville, on March 29 and 30. Professor H. J. Ettlinger of the University of Texas has accepted the invitation to be the guest speaker. The Louisiana Academy of Science will also meet at Pineville on these dates. Further details will appear in this department next month.

Problem Department

Edited by
T. A. BICKERSTAFF

This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and to solve problems here proposed.

Problems and solutions will be credited to their authors.

While it is our aim to publish problems of most interest to the readers, it is believed that regular text-book problems are, as a rule, less interesting than others. Therefore, other problems will be given preference when the space for problems is limited.

Send all communications about problems to T. A. Bickerstaff, University, Mississippi.

PROBLEMS FOR SOLUTION

No. 73. Proposed by Alexander Y. Boldyreff, University of Arizona.

Prove that

$$\frac{\sqrt{20+\sqrt{20+\sqrt{20+\ldots \text{ to } \infty}}} + \sqrt{12+\sqrt{12+\sqrt{12+\ldots \text{ to } \infty}}}}{\sqrt{6+\sqrt{6+\sqrt{6+\ldots \text{ to} \infty}}}} = 3$$

Note: Due to an error in printing, this problem was stated incorrectly in the last issue.

No. 74. Proposed by Walter B. Clarke, San Jose, California.

With the notation; I for incenter, G for centroid, K for Nagel point, produce $\overline{IG}$ to V so that $2\overline{IG} = \overline{GV}$. Draw cevians through V cutting each side into two segments and show:

Half the sum of the products of the two segments of each side is equal to the difference between half the square of the sum of the sides of the triangle and the sum of the squares of the sides.

No. 75. Proposed by Alexander Y. Boldyreff, University of Arizona.

Prove that:

$$\sqrt{PQ+(P^{m-1}-Q)\sqrt[m]{PQ+(P^{m-1}-Q)\sqrt{PQ+\ldots}}}\ \text{to}\ \infty$$
$$=\sqrt[n]{PQ+(P^{n-1}-Q)\sqrt[n]{PQ+\ldots}}\ \text{to}\ \infty$$

where n, m, are positive integers $\prec 2$, and P, Q, are real numbers.

This problem is considered especially interesting because, among other things, it enables one to express, by means of continued radicals, any integer in an infinite number of ways. For example,

$$8=\sqrt{8+7\sqrt{8+7\sqrt{8+\ldots}}}\ \text{to}\ \infty$$
$$=\sqrt[3]{40+59\sqrt[3]{40+59\sqrt[3]{40+\ldots}}}\ \text{to}\ \infty$$

No. 76. Proposed by Walter B. Clarke.

Let A, B, and H be any two vertices and the orthocenter of a triangle. From midpoint of AB draw a line parallel to the bisector of angle AHB. Show that this line cuts the perimeter of triangle into two equal parts.

No. 77. Proposed by Richard A. Miller, University of Mississippi.

A car is traveling at the rate of 60 miles per hour along a level road when a "lug" is thrown from the rear wheel after making one-sixth revolution from the lowest point. If the diameter of the wheel is 30 inches and the lug is 6 inches from the outer circumference, determine the motion, the range, and the maximum height of the lug disregarding air resistance. Investigate the case where the road is inclined at an angle θ from the horizontal.

* Owing to an unaccountable delay in the transit of Problem Department material solutions and new problems are having to be deferred to the March issue.
EDITOR AND MANAGER

Of all quantities the most primitive are the natural numbers. That a natural number is an attribute of a group of objects has long been more or less completely recognized. Indeed, for ascribing to a group of objects an attribute of number, we have a foundation perfectly similar to that for ascribing to a body an attribute of shape or an attribute of color—a quality.

From Richardson and Landis' Fundamental Conception of Modern Mathematics.

Book Reviews

Edited by
P. K. SMITH

Higher Algebra. By James McGiffert, Waverly Press, Baltimore, Maryland, 1934—XIII+338 pages.

With the background of his years of experience in teaching mathematics in Rensselaer Polytechnic Institute, Dr. McGiffert has written a new and decidedly different text. He has succeeded, by means of his individuality in methods of presentation of theory, and selection of exercises, in transfusing into the pages of this book, much of his own personality, vitality, and enthusiasm.

He has not spared himself in explaining methods in detail, and in giving many illustrative examples, to make the theory sink into the mind of the average student.

The book contains seventeen chapters, including, besides the usual material found in texts on advanced algebra, a fairly rigorous chapter on Derivatives, which are most essential as a basis of the Calculus, as well as for the determination of multiple roots.

The work on the Binomial Theorem is rigorous and interesting.

The chapter on Logarithms is made *human* by a fairly complete historic note concerning Napier and Briggs, which is illuminating.

The chapter on Infinite Series is unusually complete, and is a fitting introduction to this great subject, so important in all higher analysis.

In the chapter on the Theory of Equations, much stress is laid on graphical methods, especially on Approximation by Double Position, which is equally applicble to transcendental and algebraic equations.

The arrangement of the text, and the choice of exercises make it possible to lay out courses with different objectives.

The originality of the exercises, especially in the chapter on Probability, will appeal to many students. —Lynn Merrill.

The Differential Invariants of Generalized Spaces. By T. Y. Thomas. Cambridge, at the University Press. New York, the MacMillan Co. X+241 pp.

The book under review is a connected account of the whole subject intended for research workers and others having some knowledge of the subject. It is, however, carefully written and the actual details can be followed by one who has read the first few chapters of Veblen's Cambridge Tract on Differential Invariants.

The author makes it clear at the start that the book is devoted to differential invariants and not to differential geometry; it is not the properties of figures in space that are studied, but rather the properties of space themselves.

At the beginning of Chapter I the author lays down five postulates which serve to define an n-dimensional topological space. The affine spaces of paths, the projective spaces of paths, the Riemann spaces, spaces of distant parallelism, conformal spaces, and finally Weyl spaces are then successively defined by laying down in each case appropriate additional postulates.

Each of the spaces just referred to gives rise to its own peculiar system of differential invariants. These are taken up in order in Chapters II-IV.

In Chapter V normal coordinates are introduced and used to develop a general theory of extension.

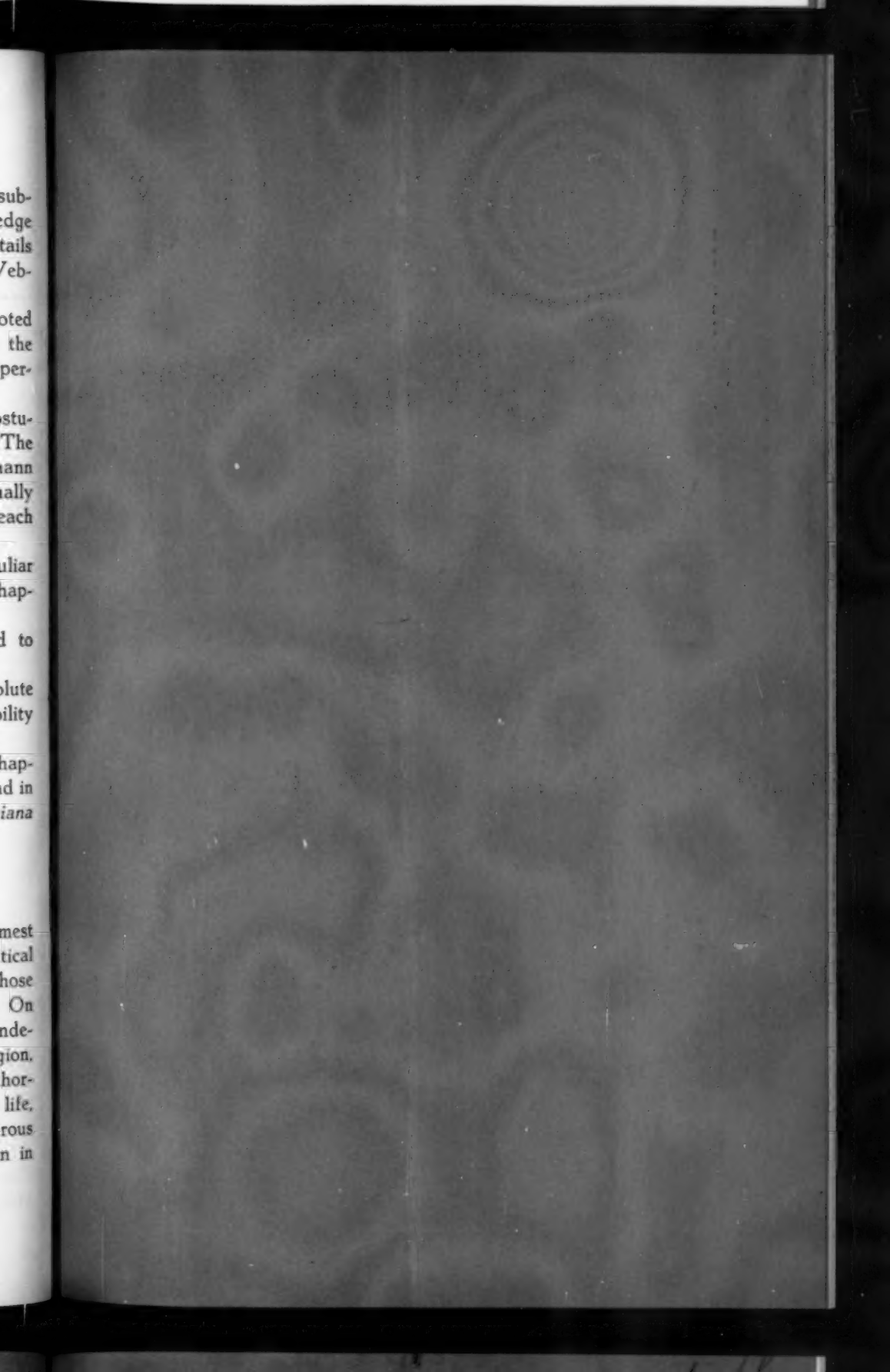
The last five chapters are devoted to spatial identities, absolute scalar differential invariants, the equivalence problems, reducibility of spaces, and functional arbitrariness of spatial invariants.

The book has an index and a bibliography at the end of each chapter. It is beautifully printed. It contains much material not to be found in any other book and should prove very useful. —H. L. Smith, *Louisiana State University.*

ON LAPLACE

That Laplace was vain and selfish is not denied by his warmest admirers; his conduct to the benefactors of his youth and his political friends was contemptible; while his appropriation of the results of those who were comparatively unknown seems to be well established. On the other side it may be said that on some questions he showed independence of character, and he never concealed his views on religion, philosophy or science, however distasteful they might be to the authoritives in power; it should also be added that towards the close of his life, and especially to the work of his pupils, Laplace was both generous and appreciative, and in one case suppressed a paper of his own in order that a pupil might have the sole credit of the investigation.

From W. W. R. Ball's Primer of the History of Mathematics.



The New Mathematics

THE FIELD of the New Mathematics covers especially the two years preceding the high school which used to be devoted to old-fashioned Arithmetic. The course also bridges the gap between these years and the first year of the high school.

IN THE New Mathematics the aim is to develop in the child by simple, progressive steps a comprehension of the significance of measurable quantity, its relation to us, and the way it can be made to serve us.

BEGINNING WITH the simple number relations which abound in the daily experiences of a child the course gradually develops the generalized number concepts of algebra, lays a foundation for the space-relations of geometry, and carries the pupil to the very threshold of the calculus.

THE NEEDS of all pupils are met whether they study mathematics for the requirements of business or professional life, or whether they seek a knowledge of mathematical relationships because of the value for culture that comes from a glimpse of absolute truth and of the practice of logical reasoning.

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